

A Rigorous and Computationally Efficient Analysis of Microstrip for Use as an Electro-Optic Modulator

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Abstract—Much interest has been shown in the literature concerning the direct modulation of an optical signal by a microwave signal making use of a microstrip-like structure with a diffused optical waveguide. Due to the particular geometry of the modulator, the usual rigorous methods of analysis, such as the spectral-domain method (SDM), encounter problems and less efficient methods have had to be used. In this paper it is shown that by using an asymptotic form of the Green's function in the standard SDM, an accurate, efficient, and rigorous full-wave analysis can readily be undertaken. A closed-form first-order solution to the field patterns is also derived.

I. INTRODUCTION

IN THE LITERATURE significant interest [1]–[6] has been shown in the direct microwave modulation of an optical signal making use of a microstrip-like structure such as that whose geometry is shown in Fig. 1. In order to optimize the performance of such a modulator it is essential to know the detailed form of the microwave electric field in the optical waveguide. Previous analyses of this structure have, however, been based either on purely static techniques such as conformal mapping [1] or the method of images [3], or, more recently, on the method of lines [5], which is in principle capable of being applied to the hybrid mode of the structure but which is computationally expensive. In [5] it is stated that the spectral-domain method (SDM), while able to give excellent results for macroscopic quantities, such as the effective permittivity, gives significant errors when used to calculate the field patterns in the vicinity of the strips. It is shown in this paper that by making several modifications to the basic spectral-domain method, it is possible to produce accurate, rigorous, and computationally efficient hybrid-mode results for the field anywhere in the structure. In fact, it is possible to take advantage of the special nature of the geometry to increase the efficiency of the computation in a way not possible with the other methods. The method can readily be applied

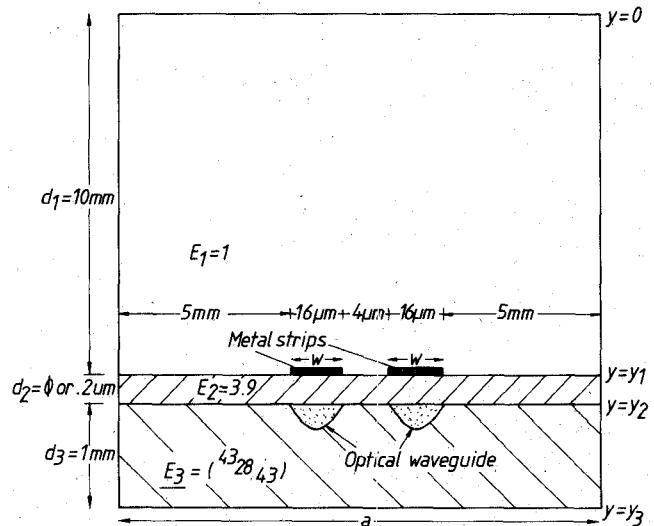


Fig. 1. Geometry of an electro-optic modulator.

to other planar structures including those with multilayer geometries.

II. THE NATURE OF THE PROBLEM

For a standard microstrip, it is possible to accurately and efficiently calculate the field pattern around the strip for the dominant and the higher order modes by means of a spectral-domain method [7]. For the geometry of Fig. 1, however, the standard spectral-domain method runs into difficulties for two reasons. First, the strip width is several orders of magnitude smaller than the box width. This means that a very large number of Fourier terms must be retained in order to achieve the required resolution around the strips. Second, the presence of the very thin buffer layer means that the Green's function for the structure converges to its asymptotic limit very slowly. Again this means that for accuracy, a very large number of terms must be taken before the asymptotic form may be used. Each of the difficulties described above can, however, be overcome in order to regain this advantage. In addition, use may be made of the fact that, given the relative sizes of the strips and the box, the effect of the box on the field in the vicinity of the strips is very small.

Manuscript received May 27, 1988; revised January 27, 1989.

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IEEE Log Number 8927798.

III. THE THEORY

In order to calculate the field pattern in the box, we must first calculate the effective permittivity of the structure using the modified spectral-domain method at the frequency of interest. Following the method used in [7], we arrive at the usual characteristic equation, the solutions of which we desire:

$$\det(K) = 0 \quad (1)$$

where

$$K_{st} = \sum_{n=0}^N \tilde{J}_s(n) \tilde{G}(n) \tilde{J}_t(n) \quad (2)$$

where N is the number of terms used in the spectral expansion. $\tilde{G}(n)$ is the dyadic Green's function for the structure, and the unknown current on the strips has been expanded in a set of basis functions, $\{\tilde{J}_s\}$.

The Green's function can be derived using the methods of [8] and [9], and the basis functions are those used in [7].

In order to facilitate the computation of the summations over n in the above equation it is useful to have an asymptotic expression for $\tilde{G}(n)$ as n goes to infinity. These are given as follows:

$$\begin{aligned} \tilde{G}_{zz}^{\infty} &= \frac{2\omega\mu}{j\alpha_n} + \frac{\beta^2 Q}{j\alpha_n \omega \epsilon_0} \\ \tilde{G}_{xz}^{\infty} &= \frac{-\beta Q}{\omega \epsilon_0} \\ \tilde{G}_{xx}^{\infty} &= \frac{j\alpha_n Q}{\omega \epsilon_0} \end{aligned} \quad (3)$$

where

$$Q = \left\{ \frac{1}{\epsilon_{G1} + \epsilon_{G2} \left\{ \frac{\epsilon_{G3} + \epsilon_{G2} \tan k_{E2} d_2}{\epsilon_{G2} + \epsilon_{G3} \tan k_{E2} d_2} \right\}} \right\}$$

$$\alpha_n = n\pi/a$$

$$k_{Hi}^2 = \epsilon_{Ti} k_0^2 - \beta^2 - \alpha_n^2$$

$$k_{Ei}^2 = \epsilon_{Ti} (k_0^2 - (\beta^2 + \alpha_n^2)/\epsilon_{Yi})$$

$$k_0^2 = \omega^2 \mu \epsilon_0.$$

Here ϵ_{Ti} is the transverse component of the permittivity of the i th layer; ϵ_{Yi} is the y component of the permittivity of the i th layer; $\epsilon_{Gi} = \sqrt{\epsilon_{Ti} \epsilon_{Yi}}$; and β is the propagation constant in the z direction.

Note that for the geometry under consideration the thickness of the middle layer is much less than the thickness of the other two layers. We therefore approximate $\tan(k_{E1} d_1)$ and $\tan(k_{E3} d_3)$ to unity but retain the

$\tan(k_{E2} d_2)$ term for the middle layer since this will converge to unity much more slowly.

We can now express the elements of the characteristic determinant in the following way:

$$\begin{aligned} \sum (\tilde{G}_{zz} - \tilde{G}_{zz}^{\infty}) \tilde{J}_z \tilde{J}_z + 2\omega\mu \sum_{n=1}^{\infty} \frac{\tilde{J}_z \tilde{J}_z}{\alpha_n} + \frac{\beta^2}{\omega \epsilon_0} \sum_{n=1}^{\infty} \frac{\tilde{J}_z \tilde{J}_z Q(n)}{\alpha_n} \\ \sum (\tilde{G}_{xz} - \tilde{G}_{xz}^{\infty}) \tilde{J}_x \tilde{J}_z - \frac{\beta}{\omega \epsilon_0} \sum_{n=1}^{\infty} \tilde{J}_x \tilde{J}_z \\ \sum (\tilde{G}_{xx} - \tilde{G}_{xx}^{\infty}) \tilde{J}_x \tilde{J}_x - \frac{1}{\omega \epsilon_0} \sum_{n=1}^{\infty} \tilde{J}_x \tilde{J}_x \alpha_n. \end{aligned} \quad (4)$$

The basis functions used to expand the unknown current are given by

$$\begin{pmatrix} J_z(x, z) \\ \frac{\partial J_x}{\partial x} \end{pmatrix} = \sum_r \begin{pmatrix} J_{zr} \\ J_{xr} \end{pmatrix} \frac{T_r(2x_r/w_r)}{(1 - (2x_r/w_r))^{1/2}} \quad (5)$$

where x_r is the displacement from the center of the r th strip; w_r is the width of the r th strip; and $T_r(x)$ are the Chebyshev polynomials.

As in [7], the Fourier transforms of these basis functions are expressed in terms of Bessel functions. By making use of the properties of the Bessel functions it is possible to evaluate the second and third terms of these expressions without the need to carry out the summations explicitly. Moreover, these terms can be evaluated once for each different geometry and thereafter need not be reevaluated. The remaining summations containing the terms

$$G_{ij} - G_{ij}^{\infty}$$

converge very rapidly and, therefore, require only a small amount of computation for their evaluation. Thus the characteristic equation for the structure can be evaluated and the effective permittivity and the current distribution in the strips calculated for any mode. This has been achieved without imposing a penalty in computational effort resulting from the large ratio of box size to strip size.

IV. EVALUATION OF THE FIELD

By making use of transmission line equivalent circuits [8], the field can be calculated by evaluating sums of the following form:

$$\begin{aligned} E_x(x, y) &= \sum_n \left\{ \tilde{G}_{xx} \tilde{J}_x + \tilde{G}_{xz} \tilde{J}_z \right\} \\ &\quad \cdot \cos \alpha_n (x + a/2) \frac{\sin k_{E1} (y_1 - y)}{\sin k_{E1} y_1}, \\ 0 < y < y_1. \end{aligned} \quad (6)$$

The other field components and at different values of y can be calculated in a similar manner.

Because of the large rate of change of field intensity in the vicinity of the strips many terms must, however, be included in the sum in order to achieve sufficient resolution. For the geometry of Fig. 1, at least 16000 terms must be included in order to achieve sufficient accuracy. This is the case whatever basis functions are used. It is this aspect of the problem which has led to the spectral-domain method being avoided hitherto. It is possible, however, to make use of the asymptotic form of the dyadic Green's function to avoid the difficulty. First, we define the asymptotic and the residual components of the field thus:

$$\begin{aligned} E_x^\infty &= \operatorname{Re} \left\{ \sum \left\{ \tilde{G}_{xx}^\infty \tilde{J}_x + \tilde{G}_{xz}^\infty \tilde{J}_z \right\} \right. \\ &\quad \left. \cdot \exp(\alpha_n((y - d_1) + j(x + a/2))) \right\} \\ E_x^{\text{res}}(x, y) &= E_x(x, y) - E_x^\infty(x, y). \end{aligned} \quad (7)$$

Similar expressions can be written for E_y and for layer 3.

With the specified basis functions, we can write E_x^∞ as the weighted sum of terms of the following form:

$$\sum_n J_s(nu) \exp(nv) \quad (8)$$

where

$$\begin{aligned} u &= \pi w/2a \\ v &= \pi(y - d_1 + jx)/a + j\pi/2 \end{aligned}$$

and s depends on which basis function we are dealing with.

For the case of a typical electro-optic modulator, to approximate the factor $Q(n)$ by a constant is no longer adequate. Instead, we make a more complicated approximation, which is valid in the case of a thin buffer layer. This is given as follows:

$$K_1 + K_2 \exp(K_3 n) \quad (9)$$

where

$$\begin{aligned} K_1 &= 1/(\epsilon_1 + \epsilon_2) \\ K_2 &= 1/(\epsilon_1 + \epsilon_3) - K_1 \\ K_3 &= a/\pi m \log(Q(m) - K_1/K_2) \\ m &= 0.55a/d_2 \epsilon_2 \pi. \end{aligned}$$

The asymptotic Green's function may still be written in terms of summations such as (8). Later, we shall require that the field pattern be normalized and we shall also need the sum

$$\sum_n \frac{J_s(nu) \exp(nv)}{n}. \quad (10)$$

As they stand, these summations converge very slowly. It is possible, however, by making use of the Laplace transforms of these expressions, to produce much more rapidly converging summations to within an additive func-

tion independent of n as follows:

$$\begin{aligned} \sum_n J_s(nu) \exp(nv) &= \sum_m \frac{1}{\left((2\pi jm + v)^2 + u^2 \right)^{1/2}} \\ &\quad \cdot \left\{ \frac{u}{2\pi jm + v + \left((2\pi jm + v)^2 + u^2 \right)^{1/2}} \right\}^s \\ \sum_n \frac{J_s(nu) \exp(nv)}{n} &= \sum_m \frac{1}{s} \left\{ \frac{u}{2\pi jm + v + \left((2\pi jm + v)^2 + u^2 \right)^{1/2}} \right\}^s, \\ s &> 0 \end{aligned}$$

$$\begin{aligned} \sum_n \frac{J_0(nu) \exp(nv)}{n} &= \sum_m \ln \left\{ \frac{u}{2\pi jm + v + \left((2\pi jm + v)^2 + u^2 \right)^{1/2}} \right\} \\ &\quad + v/2. \end{aligned} \quad (11)$$

The summations on the right converge much faster than those on the left. In fact it is often possible to obtain sufficient accuracy for practical purposes with just a single term.

The summations on the right-hand sides can be interpreted physically in the following way. The term corresponding to $m = 0$ is the only term which remains in the limit as a goes to infinity, in other words if there were no sidewalls to the structure. The other terms in the series correspond to the images produced by reflections at the sidewalls. When the strip width is very much smaller than the box width and we are only interested in the field pattern in the vicinity of the strips, then the field pattern is dominated by the singularities at the strip edges and the effect of the images is negligible in comparison. If we wished to calculate the field away from the strips then more terms would need to be taken.

In addition, because the field near the strips is dominated by the singularity at the edges of the strips, the residual component of the field is very small compared with the asymptotic component thereof. So, to a good approximation, the residual component can be neglected. We have, therefore, an easily computed closed-form expression for the field amplitude anywhere in the vicinity of the strips which can be used to predict the behavior of electro-optic components.

V. NORMALIZATION OF THE FIELD PATTERN

The solution of (4), being a homogeneous equation, will be arbitrary to within a multiplicative constant. In order to relate the field intensities to the potentials on the strips it

is therefore necessary to normalize the solutions to specified potentials. Because of the symmetry of the geometry of Fig. 1, the obvious values of the potential to use are 1 V and 1 V for the even mode and ± 1 V for the odd mode. The field pattern for any pair of potentials can then be calculated by taking a linear combination of the even and odd modes of the structure. We therefore need to impose the condition

$$\int_{-a/2}^{-(w/2 + \text{offset})} E_x(x, 0) dx = 1. \quad (12)$$

Making use of the expansion of (6), we get

$$\sum_{n=1}^{\infty} \frac{E_{xn} \sin \alpha_n (a/2 - \text{offset})}{\alpha_n} + E_{x0}(a/2 - \text{offset}) = 1 \quad (13)$$

where E_{xn} is the n th term of the summation in (6).

As before, we can split E_{xn} into its asymptotic and residual parts and thereby evaluate the left-hand side of (13) by means of a short series plus closed-form terms given in (11).

VI. COMPARISON WITH THE METHOD OF LINES

The method of lines has been applied to several planar waveguide discontinuity analyses [10]–[12] and recently to the analysis of the electro-optic modulator, which is the subject of this contribution [5]. In each case the method is used to produce an equation of the following general form:

$$\begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_z \end{pmatrix} = \begin{pmatrix} \underline{\underline{Z}}_{xx} & \underline{\underline{Z}}_{xz} \\ \underline{\underline{Z}}_{zx} & \underline{\underline{Z}}_{zz} \end{pmatrix} \begin{pmatrix} \mathbf{J}_x \\ \mathbf{J}_z \end{pmatrix}. \quad (14)$$

The vectors \mathbf{E}_x , etc., are the amplitudes of the quantity in question at a set of specified values of x along the dielectric interface containing the metallization. The matrix $\underline{\underline{Z}}$ is related to the dyadic Green's function for the structure in the following way:

$$\underline{\underline{Z}}_{ij}(n, m) = G_{ij}(x, x') \delta(x - x_n) \delta(x' - x_m) \quad (15)$$

where i and j represent x or z , and x_n and x_m are the positions of the appropriate lines.

The characteristic equation is then obtained by enforcing the boundary conditions on the strip at a number of specified test points. This yields an expression of the form

$$\det \begin{pmatrix} \underline{\underline{Z}}_{xx} & \underline{\underline{Z}}_{xz} \\ \underline{\underline{Z}}_{zx} & \underline{\underline{Z}}_{zz} \end{pmatrix} = 0 \quad (16)$$

where the size of the determinant is equal to the number of

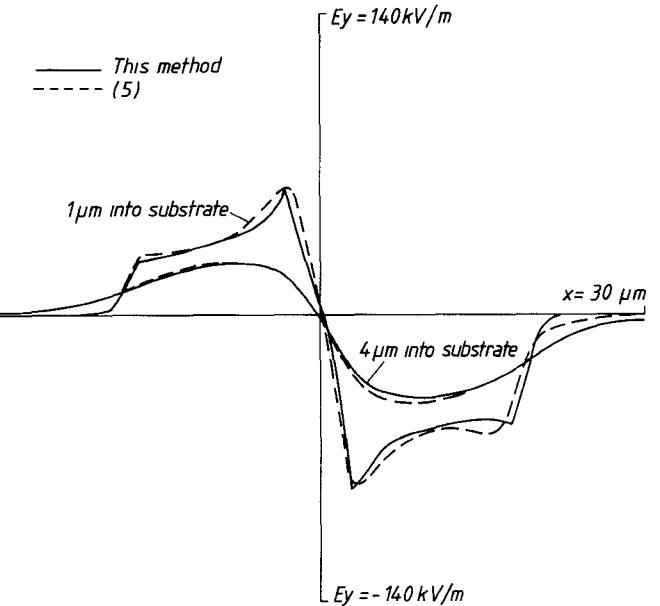


Fig. 2. Intensity of E_y for the geometry of Fig. 1 with $d_2 = 0.2 \mu\text{m}$.

test points or "lines" which are taken on the metallizations.

It can be seen that, formally, the method of lines is equivalent to the method of moments with the unknown currents being expanded in a series of impulse functions, and with the test functions also being a number of impulse functions. The distinctive feature of the method of lines is the means by which the Green's function is derived. The following observations may be made in comparison:

- i) Since the method of lines is inherently a point-matching technique, the singularities at the edges of the strips cannot be exactly taken into account. At best the test points can be concentrated into the regions near the edges. In [11] the error caused by the presence of the singularity is compensated approximately by a judicious choice of the position of the test points. With the SDM it is possible to incorporate the edge singularities directly into the basis functions.
- ii) The method of lines is likely to encounter problems, especially if higher order modes are required, due to spurious modes and relative convergence phenomena arising from the fact that the currents and fields are specified at only a finite number of points. By making use of basis functions and test functions which closely resemble the actual current distributions, the SDM does not suffer from these undesirable effects.
- iii) With the method of lines the field at any point in the box is only available as a summation in terms of the currents at the test points. With the SDM it is possible to extract a closed-form approximation because of the ability to expand the current functions in a very short series of basis functions.

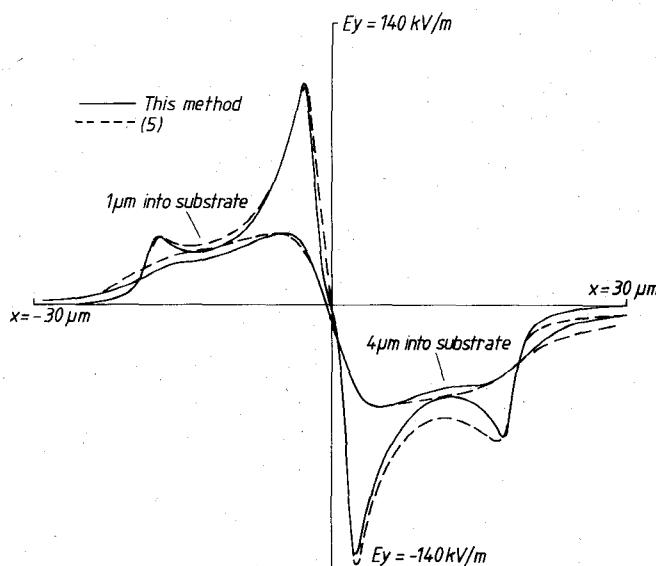


Fig. 3. Intensity of E_y for the geometry of Fig. 1 with $d_2 = 0$.

VII. THEORETICAL RESULTS

The method here described has been used to calculate the field patterns for the electro-optic modulator shown in Fig. 1 both with and without the thin buffer layer of silica. In Figs. 2 and 3 are plotted the calculated results compared with those calculated in [5] by the method of lines. In order to directly compare the results, the voltages on the strips have been set to 0 and 1 V, respectively, and the calculations were carried out at a frequency of 1 GHz, where the results would be expected to be very similar to those from a static formulation. Note that the choice of applied voltages causes an asymmetry in the field pattern. It can be seen that the results are very similar, thus indicating that each method is capable of giving accurate results. The advantage of the method described in this paper is in the computational efficiency and in the availability of a closed-form approximation for the field amplitude.

Using this method it is possible to calculate the results given in the figures using an Amstrad 1512 with coprocessor and computer times as follows: The values of effective permittivity for a given geometry and frequency were calculated in 5.5 min for each mode. Thereafter the field values at any specified point can be calculated in 0.4 s.

VIII. CONCLUSION

In this paper, it has been shown that with some modifications, the spectral-domain method can be used to analyze planar waveguide structures with a very thin buffer layer such as those being considered for use in electro-optic modulators. These have previously only been analyzed using static approximations or by means of the method of lines. The method described in this contribution is rigorous and computationally efficient and takes into account the hybrid nature of the microwave field. In addition it is capable of producing a closed-form first-order solution to

the field patterns of interest which is in a useful form for the solution of the modulator overlap integral. The results calculated using the closed-form formula derived using this method have been shown to be in good agreement with those calculated using the numerical method of lines.

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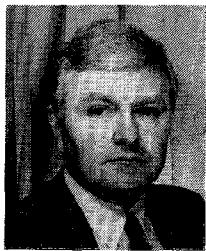
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